CALCULATION OF A TURBULENT BOUNDARY LAYER ON A PLATE IN THE CASE OF INTENSIVE INJECTION

L. N. Dymant and V. M. Spanovskii UDC 532.517.4

A method is proposed for the calculation of a turbulent layer with the use of a two-zone model with a known profile of the averaged velocity for the external zone.

A considerable number of papers is devoted to the investigation of the flow of penetrable surfaces. Turbulent boundary layers (TBL) with a porous feed of the matter in the case of moderate parameters of injection are fairly comprehensively experimentally and theoretically analyzed. At the same time, interest recently has grown in the use of strong injections in connection with a number of practical applications, in particular, for the protection of surfaces against the fall of aggressive particles, organization of gas dispersive curtains for screening of surfaces from radiative heat flows, protection of the walls of powerful MHD generators, and others. The investigations in the domain of considerable injection parameters are limited to a small number of experimental results [1-5, 7]. The use of the calculation methods worked out for moderate injections in the given case is made difficult, in connection with the fact that structural changes of the flow which occur during the action of a transverse flow of matter near the wall region of the boundary layer, such as the laminar sublayer becoming turbulent, vanishing of the friction at the wall, are important. This violates the original assumptions of the calculation methods just mentioned.

In the present paper an attempt is made to calculate TBL on a penetrable plate in the case of intensive injections, when friction at the wall is absent. In the calculation we have used the result obtained earlier [6] concerning the universality of the structure of the flow in the external "jet" zone of boundary layers with various disturbing factors, which allows us to introduce into the analysis the profile of the averaged velocity for this zone in the form

$$\frac{1-\bar{u}}{1-\bar{u}_{mm}}=\left(\frac{1-\eta}{1-\eta_{mm}}\right)^{3/2}. \tag{1}$$

This relation describes the distribution of velocity in the external zone in terms of the coordinate η_{mm} and the velocity $\bar{u}_{mm}$ on the boundary between the given zone and the internal close-to-wall zone.

In [6, 7], it is shown that in developed TBL the boundary between the zones is provided by a surface on which the shear stress has a maximum. Such a method of defining this boundary is convenient for layers with arbitrary disturbing factors and conditions at the wall, and also for layers on a nonpenetrable surface without a longitudinal pressure gradient.

It should be noted that such a method of dividing a TBL into zones was for the first time introduced by Klauser [8] for gradiential flows, and was subsequently also used for flows with injection, where experimental data on the characteristics of the flow were generalized with the use of the maximum shear stress [3, 5, 9-11]. Use of the universal profile (1) allows us to reduce the problem of calculating the TBL in the case of intense injection to the determination of the form of the profile of averaged velocities in the internal close-to-wall zone and the parameters of the boundary between the zones. At the same time, account of the influence of the conditions on the wall on the relations forming starting point of the calculation is made simpler, since they have to be determined only for the internal part of the layer.

The method being proposed is considered on an example of the problem concerned with injection of a homogeneous gas under isothermal conditions. The calculation is carried out

---

Sevastopol Instrument Building Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 37, No. 4, pp. 589-594, October, 1979. Original article submitted April 25, 1978.

0022-0841/79/3704-1137$07.50

with the use of the Boussinesq equation

$$\tau = \rho\varepsilon \frac{du}{dy}. \tag{2}$$

The distribution of τ in the internal zone for the conditions being considered can be specified using a one-dimensional approximation for the flow in the neighborhood of the wall, neglecting the derivatives with respect to the longitudinal coordinate in the equation of motion and putting $v = v_w$ [10, 12]:

$$\tau = \rho v_w u. \tag{3}$$

On the basis of the experimental data [12-15], the profile of viscosity in the close-to-wall part of the layer is approximated by a second-degree polynomial. Taking into account the boundary conditions $\varepsilon = 0$ for $y = 0$, $\varepsilon = \varepsilon_{mm}$ and $d\varepsilon/dy = 0$ for $y = y_{mm}$, we arrive at the relation

$$\frac{\varepsilon}{\varepsilon_{mm}} = \frac{2y}{y_{mm}} - \frac{y^2}{y_{mm}^2}. \tag{4}$$

At the same time, we assume that the coordinates of the maximum of turbulent viscosity ε_{mm} and the maximum shear stress τ_{mm} coincide.

To find the value ε_{mm} we use profile (1) in the form of a "velocity defect" [6]:

$$\frac{u_m - u}{v^*_{mm}} = \frac{2(1-\eta_{mm})}{3\beta}\left(\frac{1-\eta}{1-\eta_{mm}}\right)^{3/2}, \tag{5}$$

where β is a coefficient of proportionality in the expression for the length of path of blending in the external zone of the layer, $l = \beta y_m$. From (5) it follows that on the boundary of separation of the zones $(du/dy)_{mm} = v^*_{mm}/\beta y_m$, whence, according to (2),

$$\varepsilon_{mm} = \beta v^*_{mm} y_m. \tag{6}$$

Here we should note an important consequence of the universal profile (1) which consists of the fact that for the boundary layers being considered the quantity $\varepsilon_{mm}/v^*_{mm}y_m$ on the boundary of the internal and the "jet" zones is constant and equal to β. It is characteristic that this takes place also in the case of a boundary layer on a nonpenetrable surface, where the characteristic dynamic velocity is defined with respect to the shear stress of friction at the wall [16]. This confirms the advisability of use, in calculation schemes and generalized relations of TBL with injection, of characteristic velocity v^*_{mm} calculated with respect to the maximum shear stress.

From (2), with (3), (4), and (6) taken into account, there follows the profile of velocities for the internal zone

$$\bar{u} = \bar{u}_{mm}\left(\frac{\eta}{2\eta_{mm} - \eta}\right)^k, \tag{7}$$

where

$$k = \frac{\eta_{mm}}{2\beta}\sqrt{\frac{m}{\bar{u}_{mm}}}. \tag{8}$$

On the boundary between the zones the condition of joining of the velocity profiles (1) and (7) must be satisfied. From equality of the derivatives $(du/dy)_{y=y_{mm}-0} = (du/dy)_{y=y_{mm}+0}$, there follows the relation

$$\bar{u}_{mm} = 1 + \frac{2(1-\eta_{mm})}{9\beta^2}\left[m(1-\eta_{mm}) - \sqrt{9\beta^2 m + m^2(1-\eta_{mm})^2}\right], \tag{9}$$

which establishes the connection between the parameters $\bar{u}_{mm}$, η_{mm} on the boundary. For the closure of the system of equations, in the present method integral equations of impulses for the entire TBL and separately for the internal zone have been used. The first of them, for the conditions being considered, gives

$$\delta^{**} = \delta_0^{**} + mx. \tag{10}$$

To write the equation in the internal zone, we use the assumption substantiated in [8] about the profiles having the property of an approximate self-model $\bar{u} = f(y/\delta^{**})$. In this case integration of the equation of motion within the limits of the internal zone, with $d\delta^{**}/dx = m$ taken into account, leads to

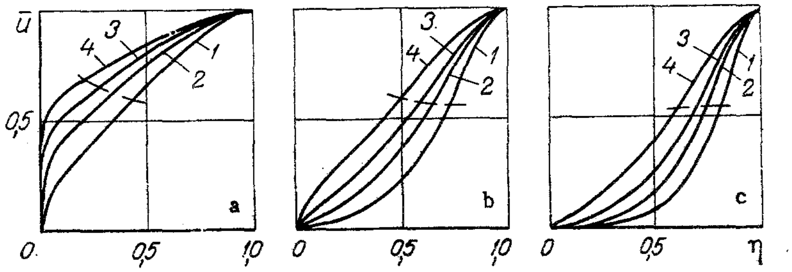

Fig. 1. Calculated velocity profiles: a) m = 0.01; b) 0.05; c) 0.10; 1) β = 0.07; 2) 0.09; 3) 0.11; 4) 0.14; dashed lines) boundaries between the close-to-wall and "jet" zones.

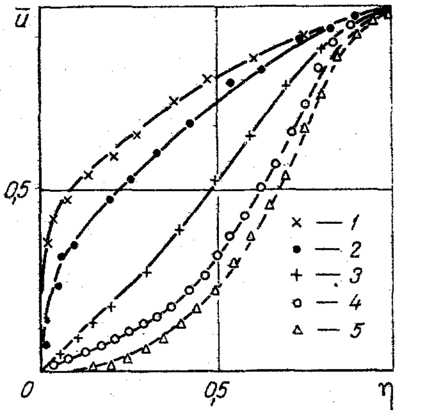

Fig. 2

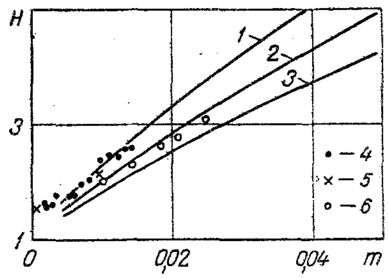

Fig. 3

Fig. 2. Comparison of calculated velocity profiles with experimental data: 1) [2], m = 0.005; 2) [19], m = 0.0095; 3) [2], m = 0.0243; 4) [4], m = 0.0576; 5) [4], m = 0.0770; solid curve) calculation with β = 0.085; dashed curve) calculation with β = 0.09.

Fig. 3. Influence of injection on the form parameter H. Solid curves) calculation; points) experiment; 1) β = 0.07; 2) 0.08; 3) 0.09; 4) [2]; 5) [19]; 6) [15], data of McQuide.

$$\bar{\tau}-\bar{\tau}_w = m\left[\bar{u}-\bar{u}\int_0^{y/\delta^{**}} \bar{u}\,d(y/\delta^{**}) + \int_0^{y/\delta^{**}} \bar{u}^2 d(y/\delta^{**})\right]. \tag{11}$$

From the condition of extremality of the shear stress on the boundary between the zones $(d\tau/dy)_{mm} = 0$, with (10) taken into account, we have

$$\int_0^{y_{mm}} \frac{u}{u_m}\,dy = \int_0^{x} \frac{v_w}{u_m}\,dx. \tag{12}$$

Relation (12) is valid if the initial thickness of the layer is zero. At the same time, it shows that for the self-model property of profiles $\bar{u} = f(y/\delta^{**})$ adopted, the longitudinal mass flow in the internal zone is equal to the mass being injected up to the section of the boundary layer under consideration. In [5], from experimental data, it is established that the "dividing line of flow" corresponds well to the coordinates of maximum shear stress, thus confirming the assumption made about the self-model property of the velocity profiles. It should be noted that in the presence of the layer in the initial section, in relation (12) we have to take into account the initial thickness of the impulse loss δ_0^{**}, while the longitudinal coordinate x is measured from the start of the injection.

As a result, the problem of an isothermal TBL on a plate with intense injection reduces to the solution of the system of equations (9), (10), and (12) with the profiles (1) and (7) taken into account.

The question concerning the choice of the value for the coefficient β is important. A survey of numerous experimental data in Soviet and foreign literature points to a considerable range of this quantity. In accordance with estimates in [17], this quantity can assume values from 0.07 to 0.14 dependent on the degree of turbulence of the external flow; in [18] the authors assume that for gradiential flows close to the separation β varies along the flow line and decreases to 0.045. Calculation in the case of different values of β shows a substantial dependence of the characteristics of TBL on the magnitude of this coefficient. In Fig. 1 we have presented calculated velocity profiles for different m and β. We see that a corresponding increase in the growth of the degree of turbulence of the external flow leads to packing of the profiles; the position of the zone boundary is shifted toward the wall, which confirms the reduction of the disturbing influence of the wall and a transformation of processes which are characteristic to free turbulence.

A comparison of the calculated velocity profiles with experimental profiles is carried out according to [2, 4, 19]. These data, obtained in aerodynamic tubes with a fairly low degree of turbulence of the flow, are in satisfactory agreement in the case of β equal to 0.085 and 0.09 (Fig. 2). From the comparison of the calculation with experimental data just presented, it is seen that the method being proposed allows us to describe the profiles in the case of moderate injection and S-formed profiles in the case of strong injection.

In Fig. 3 we have presented a comparison of the calculated dependence of the form parameter H on the intensity of injection with experimental data. The substantial scatter of experimental points of different authors does not allow us to establish the degree of correspondence between the calculated and experimental dependence; furthermore, the data existing in the literature are restricted to the injection parameter $m < 0.025$. At the same time, all experimental points lie in the region of the calculated curves for variation of the coefficient β from 0.07 to 0.09.

NOTATION

y, transverse coordinate; $\eta = y/y_m$, dimensionless transverse coordinate; x, longitudinal coordinate; u, longitudinal averaged velocity; $\bar{u} = u/u_m$, dimensionless longitudinal averaged velocity; v, transverse velocity; $v^* = \sqrt{\tau/\rho}$, dynamic velocity; τ, shear stress; $\bar{\tau} = \tau/\rho u_m^2$, dimensionless shear stress; ρ, density; l, length of merging path; δ**, thickness of impulse loss; δ*, thickness of dislodgement; $H = \delta^*/\delta^{**}$, form parameter; ε, turbulent viscosity; β, coefficient of proportionality; $m = v_w/u_m$, intensity of transverse injection. Indices: m, on the line of maximum velocity; mm, on the line of maximum shear stress; w, on the wall; 0, in the initial section.

LITERATURE CITED

1. V. P. Mugalev, Izv. Vyssh. Uchebn. Zaved., Aviats. Tekh., No. 3 (1959).
2. V. M. Polyaev, I. V. Bashmakov, D. I. Vlasov, and I. M. Gerasimov, Heat and Mass Transfer [in Russian], Vol. 1, Part 2, ITMO Akad. Nauk BSSR, Minsk (1972).
3. V. M. Polyaev, I. V. Bashmakov, D. I. Vlasov, and I. M. Gerasimov, in: Close-to-Wall Turbulent Flow [in Russian], Part 2, ITF Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1976).
4. V. M. Eroshenko, A. L. Ermakov, A. A. Klimov, V. M. Motulevich, and Yu. N. Terent'ev, Heat and Mass Transfer [in Russian], Vol. 1, Part 1, ITMO Akad. Nauk BSSR, Minsk (1972).
5. Fernandez, Zukoski, Raket. Tekh. Kosmonavtika, 7, No. 9 (1969).
6. A. N. Boiko, L. N. Dymant, and V. M. Eroshenko, Inzh.-Fiz. Zh., 27, No. 1 (1974).
7. A. N. Boiko, L. N. Dymant, V. M. Eroshenko, and V. M. Spanovskii, Inzh.-Fiz. Zh., 27, No. 4 (1974).
8. R. Klauser, Problems of Mechanics [Russian translation], No. 2, IL, Moscow (1959).
9. Mickley, Smith, Raket. Tekh. Kosmonavtika, 1, No. 7 (1963).
10. Buldridge and Muzzy, Raket. Tekh. Kosmonavtika, 4, No. 11 (1966).
11. A. D. Rekin, Inzh.-Fiz. Zh., 30, No. 6 (1976).
12. R. L. Simpson, J. Fluid Mech., 42, No. 4 (1970).
13. P. S. Andersen, W. M. Kays, and R. J. Moffat, J. Fluid Mech., 69, No. 2 (1975).
14. Galbraith and Head, Aeronaut. Q., 26, No. 2 (1975).
15. Maze and Macdonald, Raket. Tekh. Kosmonavtika, 6, No. 1 (1968).
16. I. Nikuradze, in: Problems of Turbulence [Russian translation], ONTI, Moscow—Leningrad (1936).
17. K. K. Fedyaevskii, A. S. Ginevskii, and A. V. Kolesnikov, Calculation of a Turbulent Boundary Layer of an Incompressible Fluid [in Russian], Sudostroenie, Leningrad (1973).
18. V. N. Dolgov and V. M. Shulemovich, Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1977).
19. R. L. Simpson, W. M. Kays, and R. J. Moffat, Report No. HMT-2, Stanford Univ. (1967).